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A non-commutative version of Nikishin's theorem

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Abstract

Let τ be a tracial normal state on a von Neumann algebra, $L^1(\tau)$ be the space of integrable self-adjoint operators, and S be the space of self-adjoint measurable operators. We prove that every positive linear operator from an ordered Banach space to S can be factorized through $L^1(\tau)$.

Keywords: von Neumann algebra; Tracial normal state; Measurable operator; Positive operator; Factorization of operators

In what follows, τ is a faithful normal tracial state on a (finite) von Neumann algebra M of operators in a Hilbert space H. We will denote by M^{pr} the orthomodular lattice of all orthogonal projections in M and by M_* the predual of M.

Recall some definitions and facts of the non-commutative integration (see, e.g., [5, Ch. IX, Section 2]) adapted to the case of finite trace and self-adjoint operators.

A self-adjoint operator x in H with the spectral resolution

$$x = \int_{-\infty}^{+\infty} \lambda \, de_\lambda^x$$

is said to be *affiliated* with M if $e_\lambda^x \in M^{\mathrm{pr}}$ for all $\lambda \in \mathbb{R}$. We will denote by S and S^+ the set of all self-adjoint operators affiliated with M and the subset of positive operators, respectively. For

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